

Two photon effects in electron scattering

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Abstract. The apparent discrepancy between the Rosenbluth and the polarization transfer method for the ratio of the electric to magnetic proton form factors can be explained by a two-photon exchange correction which does not destroy the linearity of the Rosenbluth plot. Though intrinsically small, of the order of a few percent of the cross section, this correction is kinematically enhanced in the Rosenbluth method while it is small for the polarization transfer method, at least in the range of Q^2 where it has been used until now.

PACS. 25.30.Bf Elastic electron scattering – 13.40.Gp Electromagnetic form factors – 24.85.+p Quarks, gluons, and QCD in nuclei and nuclear processes

1 Introduction

The electro-magnetic form factors are essential pieces of our knowledge of the nucleon structure and this justifies the efforts devoted to their experimental determination. They are defined by the matrix elements of the electro-magnetic current $J^\mu(x)$ according to¹:

$$\langle N(p') | J^\mu(0) | N(p) \rangle = e \bar{u}(p') \left[G_M(Q^2) \gamma^\mu - F_2(Q^2) \frac{p+p'}{2M} \right] u(p), \quad (1)$$

where $e \simeq \sqrt{4\pi/137}$ is the proton charge and M the nucleon mass. The magnetic form factor G_M is related to the Dirac (F_1) and Pauli (F_2) form factors by $G_M = F_1 + F_2$. Here we consider only the proton case, so we have $F_1(0) = 1$, $F_2(0) = \mu_p - 1 = 1.79$. In the one photon exchange or Born approximation, elastic lepton scattering:

$$l(k) + N(p) \rightarrow l(k') + N(p') \quad (2)$$

gives direct access to the form factors in the spacelike region ($Q^2 > 0$) where they are real. Here we adopt the usual definitions:

$$P = \frac{p+p'}{2}, K = \frac{k+k'}{2}, q = k - k' = p' - p, \quad (3)$$

and choose

$$Q^2 = -q^2, \nu = K.P \quad (4)$$

as the independent invariants of the scattering. In the Born approximation the elastic cross section is written:

$$d\sigma_B = C_B(Q^2, \varepsilon) \left[G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2) \right] \quad (5)$$

¹ The spinors satisfy $\bar{u}(p)u(p) = 2M$ and the free states are normalized as $\langle N(p') | N(p) \rangle = (2\pi)^3 2p^0 \delta(\mathbf{p} - \mathbf{p}')$.

where the electric form factor is defined by $G_E = F_1 - \tau F_2$ with $\tau = Q^2/4M^2$ and $C_B(Q^2, \varepsilon)$ is a known phase space factor which is irrelevant in what follows. The polarization parameter of the virtual photon has the expression²

$$\varepsilon = \frac{\nu^2 - M^4 \tau (1 + \tau)}{\nu^2 + M^4 \tau (1 + \tau)}, \quad (6)$$

so, at fixed Q^2 , giving ε is equivalent to give ν .

For a given Q^2 , (5) shows that it is sufficient to measure the cross section for two values of ε to determine the form factors G_M and G_E . In the following the determination of G_M and G_E using (5) will be referred to as the Rosenbluth method [1]. The fact that the combination $d\sigma/C_B(Q^2, \varepsilon)$ is a linear function of ε (Rosenbluth plot criterion) is generally considered as a test of the validity of the Born approximation. We shall see below that this criterion is not strong enough.

Polarized lepton beams give another way to access the form factors [2]. In the Born approximation, the polarization of the recoiling proton along its motion (P_l) is proportional to G_M while the component perpendicular to the motion (P_t) is proportional to G_E . We call this the polarization method for short. Because it is much easier to measure ratios of polarizations, it has been used mainly to determine the ratio G_E/G_M through a measurement of P_t/P_l for which one finds the expression [3]:

$$\frac{P_t}{P_l} = -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}} \frac{G_E}{G_M}. \quad (7)$$

So, in the framework of the Born approximation, one has two independent measurements of $R = G_E/G_M$. In Fig. 1 we show the corresponding results, which we call

² This expression assumes a negligible electron mass.

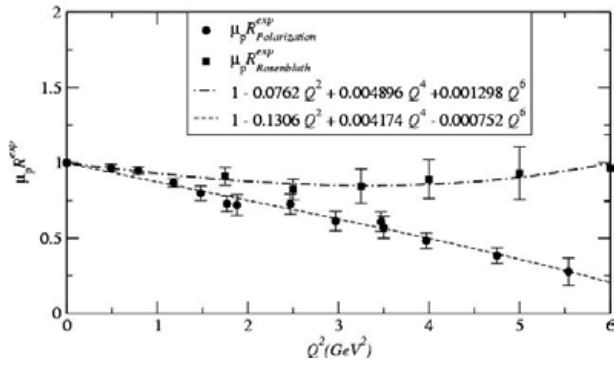


Fig. 1. Experimental values of $R_{Rosenbluth}^{exp}$ and $R_{Polarization}^{exp}$ and their polynomial fit

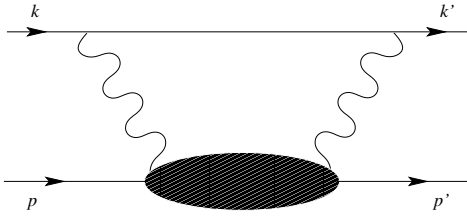


Fig. 2. The box diagram. The filled blob represents the response of the nucleon to the scattering of the virtual photon

$R_{Rosenbluth}^{exp}$ and $R_{Polarization}^{exp}$, for the range of Q^2 which is common to both methods. The data are taken from [4, 7, 8]. The deviation between the two methods starts at $Q^2 = 2 - 3 \text{ GeV}^2$ and increases with Q^2 , reaching a factor 4 at about $Q^2 = 6 \text{ GeV}^2$. This discrepancy is a serious problem as it generates confusion and doubt about the whole methodology of lepton scattering experiments.

To unravel this problem we have to give up the beloved one photon exchange concept and enter the not well paved path of multi-photon physics. By this we do not mean the effect of soft (real or virtual) photons, that is the radiative corrections. The effect of the latter is well under control because their dominant (infra-red) part can be factorized in the observables and therefore does not affect the ratio G_E/G_M . Here we must consider genuine exchange of hard photons between the lepton and the hadron. Even if we restrict to the two photon exchange case, the evaluation of the box diagram 2 involves the *full* response of the nucleon to the scattering of a virtual photon and we do not know how to perform this calculation in a model independent way. Therefore we adopt a modest strategy based on the phenomenological consequences of using the full eN scattering matrix rather than its Born approximation. Though it cannot lead to a full answer it produces the following interesting results [5]:

- the 2-photon exchange amplitude needed to explain the discrepancy is actually of the expected order of magnitude, that is a few percent of the Born amplitude.
- there may be a simple explanation to the fact that the Rosenbluth plot looks linear even though it is strongly affected by the 2-photon exchange.

- the polarization method result is little affected by the 2-photon exchange, *at least in the range of Q^2 which has been studied until now.*

2 Amplitude decomposition

The simplest way to get the general form of the (e, p) scattering amplitude is to consider its helicity matrix elements $T(h'_e h'_p; h_e h_p)$ in the center of mass $\mathbf{p} + \mathbf{k} = 0$. Due to rotational invariance T depends on $E_{cm}, \cos \theta_{cm}$ which do not change under the parity and time reversal operations. Since QED is invariant with respect to these operations, one must have

$$T(h'_e h'_p; h_e h_p) = T(-h'_e - h'_p; -h_e - h_p), \quad (8)$$

$$T(h'_e h'_p; h_e h_p) = T(h_e h_p; h'_e h'_p). \quad (9)$$

Note that these equalities generally involve a phase factor which depends on the phase convention for the helicity states. For our purposes this factor is irrelevant. One can also check that charge conjugation does not bring in additional constraints. If one applies the relations (8, 9) to the $2^4 = 16$ helicity amplitudes one finds that only 6 of them are independent. Moreover 3 of them change the electron helicity which implies that they are suppressed by an electron mass factor. So one ends with

$$T\left(\frac{1}{2} \frac{1}{2}; \frac{1}{2} \frac{1}{2}\right), T\left(\frac{1}{2} \frac{1}{2}; \frac{1}{2} - \frac{1}{2}\right), T\left(\frac{1}{2} - \frac{1}{2}; \frac{1}{2} - \frac{1}{2}\right)$$

as the only independent amplitudes. The next step is to write a covariant decomposition with 3 independent Lorentz structures. For obvious reasons we choose two of them to be the same as in the one photon approximation. For the third structure several choices are possible and we found convenient to choose

$$\bar{u}(k') \gamma_\mu P u(k) \bar{u}(p') \gamma_\nu K u(p).$$

so that the T matrix can be written as

$$T = \frac{e^2}{Q^2} \bar{u}(k') \gamma_\mu u(k) \times \bar{u}(p') \left(\tilde{G}_M \gamma^\mu - \tilde{F}_2 \frac{P^\mu}{M} + \tilde{F}_3 \frac{\gamma \cdot K P^\mu}{M^2} \right) u(p), \quad (10)$$

where $\tilde{G}_M, \tilde{F}_2, \tilde{F}_3$ are complex functions of ν and Q^2 and the factor e^2/Q^2 has been introduced for convenience. By analogy we define:

$$\tilde{G}_E = \tilde{G}_M - (1 + \tau) \tilde{F}_2 \quad (11)$$

which is equal to G_E in the Born approximation. The last step is to evaluate the matrix elements of T in the helicity basis and in the CM frame and to check that the set of equations

$$T\left(\frac{1}{2} \frac{1}{2}; \frac{1}{2} \frac{1}{2}\right), T\left(\frac{1}{2} \frac{1}{2}; \frac{1}{2} - \frac{1}{2}\right), T\left(\frac{1}{2} - \frac{1}{2}; \frac{1}{2} - \frac{1}{2}\right)$$

$$\leftrightarrow \tilde{G}_M, \tilde{F}_2, \tilde{F}_3$$

can be inverted, which is indeed the case. Note that one can also start directly from the general amplitude for elastic scattering of two spin 1/2 particles as derived by Goldberger et al. [6]. Neglecting the amplitudes which flip the helicity of the electron this amplitude is written [6]:

$$T = \bar{u}(k')\gamma.Pu(k)\bar{u}(p')(S + V\gamma.K)u(p) + A\bar{u}(k')\gamma_5\gamma.Pu(k)\bar{u}(p')\gamma_5\gamma.Ku(p) \quad (12)$$

Using Dirac equation and elementary relations among the Dirac matrices, it is then a simple exercise to write (12) in the form (10).

If one compares with the Born amplitude:

$$T_B = e^2\bar{u}(k')\gamma_\mu u(k)\frac{1}{Q^2}\bar{u}(p')\left(G_M\gamma^\mu - F_2\frac{P^\mu}{M}\right)u(p), \quad (13)$$

one gets the relations:

$$\begin{aligned} \tilde{G}_M^{Born}(\nu, Q^2) &= G_M(Q^2), \\ \tilde{F}_2^{Born}(\nu, Q^2) &= F_2(Q^2), \\ \tilde{F}_3^{Born}(\nu, Q^2) &= 0. \end{aligned} \quad (14)$$

Since \tilde{F}_3 and the phases of \tilde{G}_M and \tilde{F}_2 vanish in the Born approximation, they must originate from processes involving at least the exchange of 2-photon. If we take care of the factor e^2 introduced in the definition (13), we see that they are at least of order e^2 . This, of course, assumes that the phases of \tilde{G}_M and \tilde{F}_2 are defined, which amounts to suppose that, in the kinematical region of interest, the modulus of \tilde{G}_M and \tilde{F}_2 do not vanish. We take it for granted in the following but this restriction must be kept in mind if one goes in regions where some amplitude becomes very small.

3 Cross section and polarization transfer

If we define:

$$\begin{aligned} \tilde{G}_M &= e^{i\phi_M} |\tilde{G}_M|, \\ \tilde{G}_E &= e^{i\phi_E} |\tilde{G}_E|, \\ \tilde{F}_i &= e^{i\phi_i} |\tilde{F}_i|, \end{aligned} \quad (15)$$

then, using standard techniques, we get the following expressions for the observables of interest:

$$\begin{aligned} \frac{d\sigma}{C_B(\varepsilon, Q^2)} &= |\tilde{G}_M|^2 + \frac{\varepsilon}{\tau} |\tilde{G}_E|^2 \\ &+ 2\varepsilon\rho\mathcal{R} \left(\left(\tilde{G}_M + \frac{1}{\tau}\tilde{G}_E \right) \tilde{F}_3^* \right) \\ &+ \left(\frac{1}{\tau} + \frac{2\varepsilon}{1+\varepsilon} \right) \varepsilon\rho^2 |\tilde{F}_3|^2, \end{aligned} \quad (16)$$

$$\frac{P_t}{P_l} = -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}} \frac{|\tilde{G}_E| \cos\phi_{ME} + \rho |\tilde{F}_3| \cos\phi_{3M}}{|\tilde{G}_M| + \frac{2\varepsilon}{1+\varepsilon}\rho |\tilde{F}_3| \cos\phi_{3M}}, \quad (17)$$

where:

$$\phi_{ME} = \phi_M - \phi_E, \quad \phi_{3M} = \phi_3 - \phi_M, \quad \rho = \frac{\nu}{M^2}. \quad (18)$$

If one substitutes the Born approximation values of the amplitudes (14) then (16, 17) give back the familiar expressions (5, 7).

To simplify the general expressions (16, 17) we make the very reasonable assumption that only the two photons exchange needs to be considered. This amounts to keep only the terms of order e^2 with respect to the leading one in (16, 17). Using the fact that ϕ_M , ϕ_E and \tilde{F}_3 are of order e^2 we get the approximate expressions:

$$\begin{aligned} \frac{d\sigma}{C_B(\varepsilon, Q^2)} &= |\tilde{G}_M|^2 \\ &\times \left\{ 1 + \frac{\varepsilon}{\tau} \frac{|\tilde{G}_E|^2}{|\tilde{G}_M|^2} + 2\varepsilon \left(1 + \frac{1}{\tau} \frac{|\tilde{G}_E|}{|\tilde{G}_M|} \right) Y_{2\gamma} \right\} \end{aligned} \quad (19)$$

and

$$\begin{aligned} \frac{P_t}{P_l} &= -\sqrt{\frac{2\varepsilon}{\tau(1+\varepsilon)}} \\ &\times \left\{ \frac{|\tilde{G}_E|}{|\tilde{G}_M|} + \left(1 - \frac{2\varepsilon}{1+\varepsilon} \frac{|\tilde{G}_E|}{|\tilde{G}_M|} \right) Y_{2\gamma} \right\} \end{aligned} \quad (20)$$

To set a scale for the size of the two photon correction we have introduced the dimensionless ratio:

$$Y_{2\gamma}(\nu, Q^2) = \mathcal{R} \left(\frac{\nu\tilde{F}_3}{M^2|\tilde{G}_M|} \right)$$

which should be a good measure of the effect since, if we neglect $|\tilde{G}_E|/|\tilde{G}_M|$ with respect to τ in (19), we see that

the cross section would be of the form $|\tilde{G}_M|^2 (1 + Y_{2\gamma})^2$. Therefore we expect $Y_{2\gamma}$ to be of the order of $\alpha \simeq 1/137$.

The equations (19,20) already exhibit the solution to our problem. In the cross section the ratio $|\tilde{G}_E|/|\tilde{G}_M|^2$ comes with a term $2\left(\tau + |\tilde{G}_E|/|\tilde{G}_M|\right)Y_{2\gamma}$ which at large Q^2 is essentially $2\tau Y_{2\gamma} = Q^2 Y_{2\gamma}/2M^2$. This produces an amplification of the two photon effect which is not present in P_t/P_l . As a rough estimate let us take $|\tilde{G}_E|/|\tilde{G}_M| \sim G_E(0)/G_M(0) = 1/2.79$ and choose $Q^2 = 4M^2$. Then the coefficient of ε in (19), which is supposed to measure G_E/G_M in the Born approximation, is equal to $0.128 + 2.7Y_{2\gamma}$. So even if $Y_{2\gamma}$ is as small as 1% it produces a relative correction of 21%! By contrast if we do the same for the expression in parenthesis in (20), with $\varepsilon = 0.8$ which is a typical value used in [7, 4], we get $0.36 + 0.68Y_{2\gamma}$. For $Y_{2\gamma} = 1\%$ this only produces a 2% correction. Now that the origin of the discrepancy has been identified we can try to analyze the data in a more quantitative manner.

4 Analysis

From (19,20) we see that the experimental couple $(d\sigma, P_t/P_l)$ depends on $|\tilde{G}_M|$, $|\tilde{G}_E|$ and $\mathcal{R}(\tilde{F}_3)$. In first approximation we know that

$$|\tilde{G}_M(\nu Q^2)| \simeq G_M(Q^2), \quad |\tilde{G}_E(\nu Q^2)| \simeq G_E(Q^2)$$

so only $\mathcal{R}(\tilde{F}_3)$ is really a new unknown parameter. If we look at the data of [8] for $\sigma/C_B(\varepsilon, Q^2)$ as a function of ε we observe that for each value of Q^2 the set of points are pretty well aligned. We see on (19) that this can be understood if, at least in first approximation, the product $\nu \tilde{F}_3$ is independent of ε . We do not have a first principle explanation for this but we feel allowed to take it as an experimental evidence. To explain the linearity of the plots one should also suppose that $|\tilde{G}_M|$ and $|\tilde{G}_E|$ are independent of ε (that is ν) but since the dominant term of these amplitudes depends only on Q^2 this is a very mild assumption. We then see from (19) that what is measured using the Rosenbluth method is:

$$(R_{Rosenbluth}^{exp})^2 = \frac{|\tilde{G}_E|^2}{|\tilde{G}_M|^2} + 2 \left(\tau + \frac{|\tilde{G}_E|}{|\tilde{G}_M|} \right) Y_{2\gamma}, \quad (21)$$

with $|\tilde{G}_E|/|\tilde{G}_M|$ and $Y_{2\gamma}$ essentially independent of ε , rather than

$$(R_{Rosenbluth}^{exp})^2 = \left(\frac{G_E}{G_M} \right)^2, \quad (22)$$

as implied by the one photon exchange approximation.

On the other hand the experimental results of the polarization method have been obtained for a narrow range of ε , typically ³ from $\varepsilon = .6$ to $.9$. So, in practice, we can neglect the ε dependence of $R_{Polarization}^{exp}$ and from (20) we see that this experimental ratio must be interpreted as:

$$R_{Polarization}^{exp} = \left| \frac{\tilde{G}_E}{\tilde{G}_M} \right| + \left(1 - \frac{2\varepsilon}{1+\varepsilon} \left| \frac{\tilde{G}_E}{\tilde{G}_M} \right| \right) Y_{2\gamma}, \quad (23)$$

rather than

$$R_{Polarization}^{exp} = \frac{G_E}{G_M}. \quad (24)$$

In order that (23) be consistent with our hypothesis we should find that $Y_{2\gamma}$ is small enough that the factor $2\varepsilon/(1+\varepsilon)$ introduces no noticeable ε dependence in $R_{Polarization}^{exp}$. We have now a system of (21, 23) for $|\tilde{G}_E|/|\tilde{G}_M|$ and $Y_{2\gamma}$ that we can solve for each value of Q^2 . Due to the kinematical enhancement of the two photons effect in the

³ except at the lowest values of Q^2 , where there is anyway no discrepancy between the Rosenbluth and the polarization method.

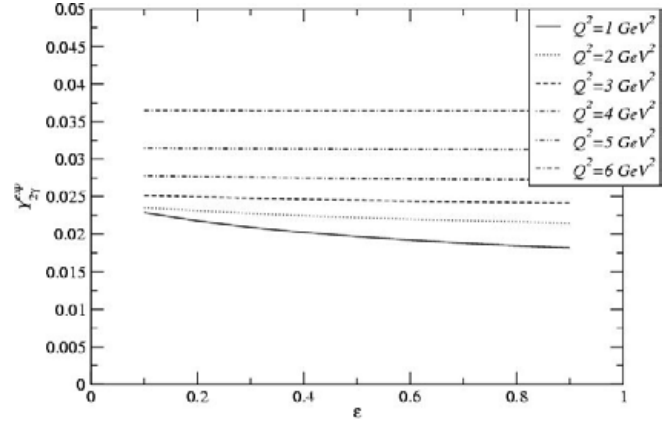


Fig. 3. The ratio $Y_{2\gamma}^{exp}$ versus ε for several values of Q^2

cross section we cannot treat it as a perturbation when solving the system of equations. Since the latter is equivalent to a quartic equation it is more efficient to solve it numerically. For this we have fitted the data by a polynomial in Q^2 . The result is shown on Fig. 1 and we shall consider this fit as the experimental values. In particular we do not attempt to represent the effect of the error bars which can be postponed to a more complete re-analysis of the data.

Using the fit we solve numerically the system (21, 23) for the couple $(Y_{2\gamma}^{exp}, R_{1\gamma+2\gamma}^{exp} = |\tilde{G}_E|/|\tilde{G}_M|)$. The solution for the ratio $Y_{2\gamma}^{exp}$ is shown on Fig. 3 where we can see that it is actually small, of the order of a few percents. Also we observe that it is essentially flat as a function of ε which is consistent with our hypothesis. In fact it is a direct consequence of the smallness of $Y_{2\gamma}^{exp}$ which multiplies the only factor which depends on ε in (23). For the same reason $R_{1\gamma+2\gamma}^{exp}$ is also essentially independent of ε .

The above result for $Y_{2\gamma}^{exp}$ indicates that the corrections to the Born approximation are actually small in absolute value. In the Rosenbluth method their effect is accidentally amplified but there is no reason to think that this kind of accident will also occur in $\tilde{G}_E - G_E$ or $\tilde{G}_M - G_M$. So it makes sense to compare the value we get for $R_{1\gamma+2\gamma}^{exp}$ with the starting experimental ratios $R_{Rosenbluth}^{exp}$ and $R_{Polarization}^{exp}$. This is shown on Fig. 4 where we see that⁴, as expected, $R_{1\gamma+2\gamma}^{exp}$ is close to $R_{Polarization}^{exp}$.

5 Conclusion

Within the hypothesis of our analysis, we come to the conclusion that the data for G_E/G_M from the Rosenbluth and the polarization method are compatible once the exchange of 2-photon is allowed in the analysis. The two photon effect is intrinsically small, as it should, but is strongly amplified in the Rosenbluth method. Assuming that the difference between G_E/G_M and \tilde{G}_E/\tilde{G}_M is of the same order as $Y_{2\gamma}^{exp}$ one can consider that the value

⁴ The calculation here has been done (arbitrarily) at $\varepsilon = 0.6$ but the result is essentially independent of ε .

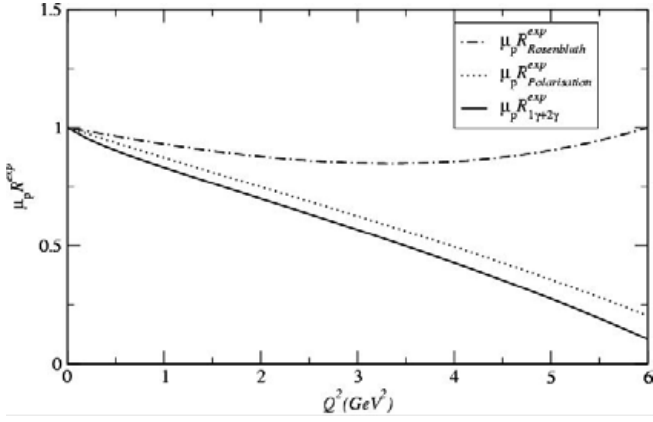


Fig. 4. Comparison of the experimental ratios $\mu_p R_{Rosenbluth}^{exp}$ and $\mu_p R_{Polarization}^{exp}$ with the value of $\mu_p R_{1\gamma+2\gamma}^{exp} = \mu_p \left| \tilde{G}_E \right| / \left| \tilde{G}_M \right|$ deduced from our analysis

we obtain for \tilde{G}_E/\tilde{G}_M is essentially the value of G_E/G_M obtained in the Born approximation but corrected by the two photon effect due to \tilde{F}_3 . Our conclusion is then that the correction which must be applied to the results of the polarization method is negligible while it is huge in the case of the Rosenbluth method.

To confirm our results one needs some model calculation of the central quantity of our analysis, namely \tilde{F}_3 . An explicit calculation of the two photon effect has been performed in [9] and it indicates that it can explain part of the discrepancy. However this calculation limits the intermediate states to the nucleon itself, which is certainly not a realistic hypothesis. A more ambitious calculation, where the intermediate states excitations are implicitly taken into account through the use of generalized parton distributions, is now close to completion [10]. The preliminary results are in agreement with our analysis.

Another important point is the study of observables which can directly test our understanding of the two photon effects. As an example we consider the beam charge asymmetry BCA , which is defined by:

$$\frac{\sigma(\text{positron})}{\sigma(\text{electron})} = 1 - 2BCA.$$

In the 2-photon approximation one gets

$$BCA = 2 \frac{\varepsilon G_E \mathcal{R}(\delta \tilde{G}_E) + \tau G_M \mathcal{R}(\delta \tilde{G}_M) + \varepsilon G_M (G_E + \tau G_M) Y_{2\gamma}}{\varepsilon G_E^2 + \tau G_M^2},$$

which in the limit $G_E/(\tau G_M) \rightarrow 0$ implies

$$\frac{\sigma(+)}{\sigma(-)} \simeq 1 - 4\varepsilon Y_{2\gamma} - 4 \frac{\mathcal{R}(\delta \tilde{G}_M)}{G_M}.$$

According to our analysis we have $Y_{2\gamma} > 0$, while the scarce existing data [11] at large Q^2 indicate that $\sigma(+)/\sigma(-)$ is compatible with one or even a bit larger. This would imply that $\delta \tilde{G}_M/G_M$ is negative and of the order of $Y_{2\gamma}$, which is of course in line with our working hypothesis. This illustrates how dedicated beam charge asymmetry experiments could be used to strengthen our understanding of two photon effects in electron scattering.

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